

# Set Theory Homework 1

1.  $(x \subseteq y) \leftrightarrow (x \cap y) = x$

2. Suppose  $A \cup B = C \cup D$ . Find  $P, Q, R$  and  $S$  such that  $A = P \cup Q$ ,  $B = R \cup S$ ,  $C = P \cup R$  and  $D = Q \cup S$ .

3. Let  $A, B$  and  $C$  be sets so that  $A$  and  $B$  are non-empty. Show that if

$$(A \times B) \cup (B \times A) = C \times C,$$

then

$$A = B = C.$$

4. Prove that a total ordering  $(A, \leq)$  is a well-ordering if and only if there is no function  $f : A \rightarrow A$  such that the following three conditions hold

(a)  $f(x) \leq x$  for all  $x \in A$

(b)  $f(x) < x$  for some  $x \in A$

(c)  $f(f(x)) < f(x)$  whenever  $f(x) < x$  and  $x \in A$ .

5. A total ordering  $(X, \leq)$  is *scattered* if there is no infinite subset  $Y$  of  $X$ , which is densely ordered. Show that if  $A$  and  $B$  are scattered subsets of a total ordering  $(X, \leq)$ , then  $A \cup B$  is also scattered. (A subset  $Y$  of a total order is *densely ordered* if for all  $a, b$  in  $Y$  such that  $a < b$  there is  $c$  in  $Y$  such that  $a < c$  and  $c < b$ .)