

Set Theory home work 6

1. Show that the cardinality of the set of all real numbers is 2^{\aleph_0} . You can identify reals with their decimal (or binary) expansions.
2. Solve the equation system
$$\begin{cases} \aleph_1 \cdot \kappa + \aleph_2 \cdot \lambda = \aleph_2 \\ \aleph_2 \cdot \kappa + \aleph_3 \cdot \lambda = \aleph_2 \end{cases}$$
 in the arithmetic of cardinal numbers, that is, find all pairs (κ, λ) of cardinals that satisfy this pair of equations.
3. Use König's Theorem to show that for infinite cardinals $\text{cf}(\lambda^\kappa) > \kappa$.
4. Prove $\aleph_5^{\aleph_0} = \aleph_5 \cdot 2^{\aleph_0}$
5. Show that there is an uncountable set $S \subseteq P(\omega)$ such that the intersection of any two distinct elements of S is finite.