

Set Theory home work 7

1. Show that the preimage of a Borel set in a continuous mapping is Borel.
2. Show that the set of real numbers in $[0,1]$ the decimal expansion of which has infinitely many digits 7 is Borel.
4. Show that
 - a. $\{\alpha : \alpha \text{ is a limit of limit ordinals } < \omega_1\}$ is club
 - b. $\{\alpha \cdot 2 : \alpha \text{ is an infinite ordinal } < \omega_1\}$ is not club.
5. Show that if $f : \omega_1 \rightarrow \omega_1$ is any function, then $\{\alpha : \forall \beta < \alpha (f(\beta) < \alpha)\}$ is club.
6. Show that if S is a stationary subset of ω_1 , then for all $\alpha < \omega_1$ there is a closed subset $S' \subseteq S$ such that $(\alpha, <) \cong (S', <)$. Here $<$ is the usual ordering of ω_1 . Hint: Use induction on α .