

Set Theory problem session 5

1. Show that every closed interval $[a,b]$ is Borel.
2. Show that the set of irrational numbers is Borel.
4. Show that
 - a. $\{\alpha : \alpha \text{ is a limit ordinal } < \omega_1\}$ is club
 - b. $\{\alpha+1 : \alpha \text{ is an ordinal } < \omega_1\}$ is not club.
5. Show that if C is club then the set of ordinals $< \omega_1$ that are limits of points of C , is club.
6. Show that if S is an uncountable set of finite subsets of ω_1 , then there is an uncountable subset S^* of S such that the sets in S^* have the same intersection. Hint: Let S consists of s_α , $\alpha < \omega_1$. If α is a limit ordinal $< \omega_1$, let $t_\alpha = \alpha \cap s_\alpha$. Now use Pressing Down Lemma to find uncountable many α with the same t_α , say t . Then finally find uncountable many s_α with the same intersection t .
7. Show that if $\bigcup_{n < \omega} S_n$ is stationary, then some S_n is stationary.