

**CAPITA SELECTA: MODEL THEORY, AXIOMATIC SET THEORY**

**TOPIC FOR 2009: DESCRIPTIVE SET THEORY**

ASSIGNMENT 2 (DUE 20 OCTOBER 2009)

- (a) Code finite sequences  $\sigma$  of numbers into numbers  $\langle\sigma\rangle$  as usual (via prime decomposition). Then a set of strings can be coded into an infinite (binary) sequence:  $\beta \in 2^\omega$  represents the set:

$$\langle\beta\rangle := \{\sigma \mid \beta(\langle\sigma\rangle) = 1\}.$$

- Show that the relation ‘ $\beta$  represents a tree’ is  $\Pi_1^0$ .
  - Show that the relation ‘ $\beta$  represents a tree  $T$  and  $\alpha$  is a path through  $T$ ’ is  $\Pi_1^0$ .
  - Show that the relation ‘ $\beta$  represents a finite branching tree’ is  $\Pi_3^0$ .
  - Show that the relation ‘ $\beta$  represents a perfect tree (i.e. with no isolated infinite paths)’ is  $\Pi_3^0$ .
- (b) Show that every  $\Sigma_n^0$  relation is  $\Sigma_n^0$  in a real parameter, and every  $\Pi_n^0$  relation is  $\Pi_n^0$  in a real parameter.
- (c) Draw the tree  $T_\alpha$  for the case where  $\alpha$  codes a  $\Sigma_n^0$  or  $\Pi_n^0$  set for  $n \leq 3$ .