CAPITA SELECTA: MODEL THEORY, AXIOMATIC SET THEORY

TOPIC FOR 2009: DESCRIPTIVE SET THEORY

ASSIGNMENT 6 (DUE 7TH OF DECEMBER 2009)

(a) Show that the sets of the form $G_{\mathbb{N}} y F(x, y)$, where $F \subseteq \mathcal{N} \times \mathcal{N}$ is closed, are exactly the $\mathbf{\Sigma}^1_1$ subsets of $\mathcal{N}$. Show that the sets of the form $G_{\mathbb{N}} y C(x, y)$, where $C \subseteq \mathcal{N} \times \mathcal{N}$ is clopen, are exactly the Borel subsets of $\mathcal{N}$.
Hint: The first part is not very hard. For the second part, let $A \subseteq \mathcal{N}$ be Borel and find F, H closed in $\mathcal{N} \times \mathcal{N}$ with $x \in A \iff \exists u,\ (x, u) \in F$ and $x \notin A \iff \exists v\ (x, v) \in H$. Let $(x, y) \in F' \iff (x, (y)_0) \in F$, $(x, y) \in H' \iff (x, (y)_1) \in H$, where for $y \in \mathcal{N}$, $(y)_0(n) = y(2n)$, and $(y)_1(n) = y(2n + 1)$. Let $C \subseteq \mathcal{N} \times \mathcal{N}$ be clopen separating F', H'. Then $x \in A \iff G_{\mathbb{N}} y C(x, y)$.

(b)