CAPITA SELECTA: MODEL THEORY, AXIOMATIC SET THEORY

TOPIC FOR 2009: DESCRIPTIVE SET THEORY

ASSIGNMENT 7 (DUE 14TH OF DECEMBER 2009)

(a) Give a direct proof that $\mathbf{\Sigma}^0_2$ games are determined as follows: Let $X \subseteq A^{\mathbb{N}}$ be $\mathbf{\Sigma}^0_2$, so that $X = \cup_n F_n$, $F_n \subseteq A^{\mathbb{N}}$ closed. Let T_n be a pruned tree with $F_n = [T_n]$. Define by transfinite recursion $W^\xi \subseteq A^{<\mathbb{N}}$ by:

$$s \in W^0 \iff \mathit{length}(s) \in 2\mathbb{N} \text{ and } \exists n \text{ (I has win. strat. in } (F_n)_s),$$

If $W^\eta, \eta < \xi$ have been defined, let

$$x \in C^{\xi,n} \iff \forall k \in 2\mathbb{N}\ (x \restriction k \in \cup_{\eta<\xi} W^\eta \cup T_n)$$

and put

$$s \in W^\xi \iff \mathit{length}(s) \in 2\mathbb{N} \text{ and } \exists n \text{ (I has win. strat. in } (C^{\xi,n})_s).$$

Show that $C^{\xi,n}$ is closed. Then show that

(1) $s \in \cup_\xi W^\xi \Rightarrow$ I has a winning strategy in X_s.
(2) $\emptyset \notin \cup_\xi W^\xi \Rightarrow$ II has a winning strategy in X.